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International Baccalaureate
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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	6

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12 pages
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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

    The following diagram shows a circle with centre O and radius 5 cm.

    diagram not to scale
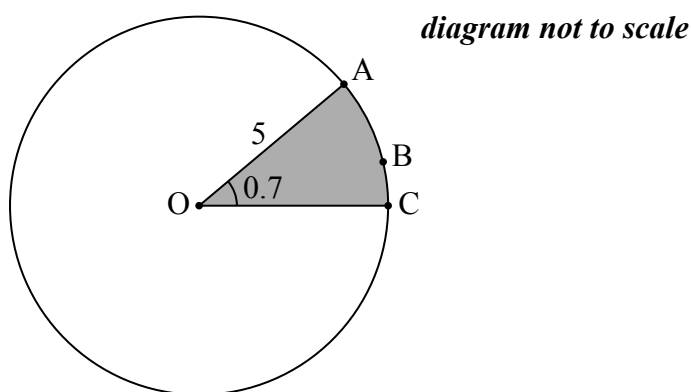
    The points A, B and C lie on the circumference of the circle, and $\hat{AOC} = 0.7$ radians .

    (a) (i) Find the length of the arc ABC.

    (ii) Find the perimeter of the shaded sector. *[4]*

    (b) Find the area of the shaded sector. *[2]*

    (This question continues on the following page)

(Question 1 continued)

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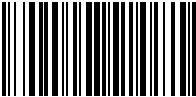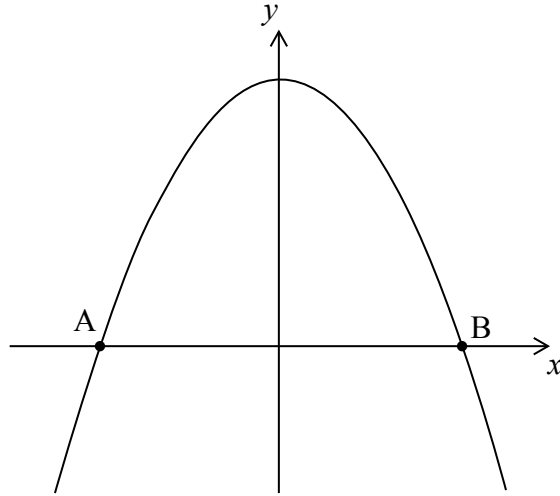
2.   *[Maximum mark: 6]*

Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.

The graph crosses the *x*-axis at the points A and B.

(a)   Find the *x*-coordinate of A and of B.   *[3]*

(b)   The region enclosed by the graph of f and the *x*-axis is revolved $360°$ about the *x*-axis.
      Find the volume of the solid formed.   *[3]*

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3. *[Maximum mark: 5]*

The following table shows the amount of fuel (y litres) used by a car to travel certain distances (x km).

Distance (x km)	40	75	120	150	195
Amount of fuel (y litres)	3.6	6.5	9.9	13.1	16.2

This data can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Write down the value of a and of b.

(ii) Explain what the gradient a represents. *[3]*

(b) Use the model to estimate the amount of fuel the car would use if it is driven 110 km. *[2]*
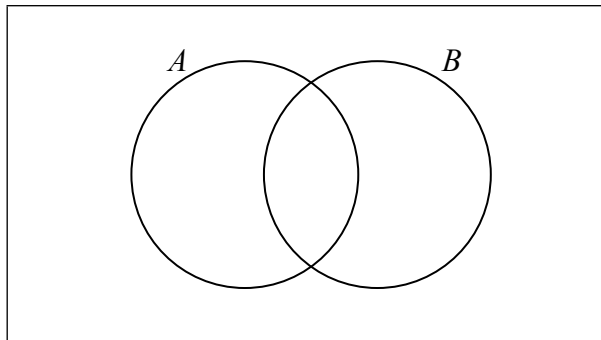
Turn over

4. *[Maximum mark: 7]*

Let A and B be independent events, where $P(A) = 0.3$ and $P(B) = 0.6$.

(a) Find $P(A \cap B)$. *[2]*

(b) Find $P(A \cup B)$. *[2]*

(c) (i) On the following Venn diagram, shade the region that represents $A \cap B'$.

(ii) Find $P(A \cap B')$. *[3]*

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5.   *[Maximum mark: 7]*

In triangle ABC, $AB = 6\,cm$ and $AC = 8\,cm$. The area of the triangle is $16\,cm^2$.

(a)   Find the two possible values for $\hat{A}$.   *[4]*

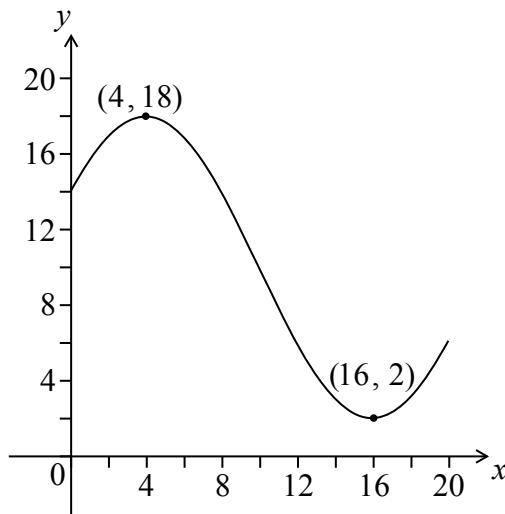(b)   Given that $\hat{A}$ is obtuse, find BC.   *[3]*

6. *[Maximum mark: 8]*

Let $f(x) = p\cos(q(x+r)) + 10$, for $0 \le x \le 20$. The following diagram shows the graph of f.

The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

(a) Write down the value of r. *[2]*

(b) (i) Find p.

(ii) Find q. *[4]*

(c) Solve $f(x) = 7$. *[2]*

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7. *[Maximum mark: 7]*

Consider the expansion of $x^2 \left(3x^2 + \dfrac{k}{x} \right)^8$. The constant term is $16\,128$.

Find k.

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Turn over

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after t hours is modelled by the function $A(t) = 12e^{0.4t}$.

(a) Find the initial number of bacteria in colony A. *[2]*

(b) Find the number of bacteria in colony A after four hours. *[3]*

(c) How long does it take for the number of bacteria in colony A to reach 400? *[3]*

The number of bacteria in colony B after t hours is modelled by the function $B(t) = 24e^{kt}$.

(d) After four hours, there are 60 bacteria in colony B. Find the value of k . *[3]*

(e) The number of bacteria in colony A first exceeds the number of bacteria in colony B after n hours, where $n \in \mathbb{Z}$. Find the value of n . *[4]*

*Do **NOT** write solutions on this page.*

9.   *[Maximum mark: 15]*

A particle moves in a straight line. Its velocity, $v\,\text{ms}^{-1}$, at time t seconds, is given by

$$v = (t^2 - 4)^3, \text{ for } 0 \le t \le 3.$$

(a)   Find the velocity of the particle when $t = 1$.   *[2]*

(b)   Find the value of t for which the particle is at rest.   *[3]*

(c)   Find the total distance the particle travels during the first three seconds.   *[3]*

(d)   Show that the acceleration of the particle is given by $a = 6t\,(t^2 - 4)^2$.   *[3]*

(e)   Find all possible values of t for which the velocity and acceleration are both positive or both negative.   *[4]*

Turn over

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

A forest has a large number of tall trees. The heights of the trees are normally distributed with a mean of 53 metres and a standard deviation of 8 metres. Trees are classified as giant trees if they are more than 60 metres tall.

(a) A tree is selected at random from the forest.

   (i) Find the probability that this tree is a giant.

   (ii) Given that this tree is a giant, find the probability that it is taller than 70 metres. *[6]*

(b) Two trees are selected at random. Find the probability that they are both giants. *[2]*

(c) 100 trees are selected at random.

   (i) Find the expected number of these trees that are giants.

   (ii) Find the probability that at least 25 of these trees are giants. *[6]*